35.6 A $5000 lb_m$ machine rotates at $1800 rpm$ and is mounted on vibration isolators with a combined stiffness of $40,000 \frac{lb_f}{in}$. In parallel with the springs, a damper has been included. The damping ratio is 0.3. An unbalanced force of $2000 lb_f$ is caused by the machine. What is the maximum force transmitted through the base?

A. $400 lb_f$

B. $900 lb_f$

C. $1100 lb_f$

D. $1400 lb_f$

Start by finding the natural frequency of the machine which is a function of the total combined spring stiffness and the mass. Note that g_c must be included to make the units consistent.

$$\omega_n = \sqrt{\frac{k g_c}{m}} = \sqrt{\frac{\left(40,000 \frac{lb_f}{in}\right)\left(12 \frac{in}{ft}\right)\left(32.2 \frac{lb_m \cdot ft}{lb_f \cdot s^2}\right)}{5,000 lb_m}} = 55.6 \frac{rad}{s}$$

Find the forcing frequency, which is a function of the rotational speed.

$$\omega = \left(1800 \frac{rev}{min}\right)\left(\frac{1 min}{60 s}\right)\left(\frac{2\pi rad}{rev}\right) = 188.5 \frac{rad}{s}$$

Find the frequency ratio, r.

$$r = \frac{\omega}{\omega_n} = \frac{188.5 \frac{rad}{s}}{55.6 \frac{rad}{s}} = 3.39$$

Determine the vibration transmissibility, $\frac{F_T}{F_o}$, where F_T is the transmitted force and F_o is the unbalanced force. The damping ratio is given as $\varsigma = 0.3$.

$$\frac{F_T}{F_o} = \left[\frac{1 + (2\varsigma r)^2}{(1 - r^2)^2 + (2\varsigma r)^2}\right]^{\frac{1}{2}} = \left[\frac{1 + [2(0.3)(3.39)]^2}{\left(1 - (3.39)^2\right)^2 + [2(0.3)(3.39)]^2}\right]^{\frac{1}{2}} = 0.21$$

Solve for the transmitted force.

$$F_T = (0.21) F_o = (0.21)(2000 lb_f) = 424 lb_f$$

Answer A